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An Introduction to Climate Modeling

Milestone 3 - Solving a zero-dimensional Energy Balance Model

1 Solving a zero-dimensional Energy Balance Model

In the previous milestones you have already implemented some routines that are part of an energy balance model. Now you have the opportunity to combine your results into a zero-dimensional ($0D$) energy balance model (EBM) and solve the model using an ODE solver. Note that for zero-dimensional EBMs, spatial dependence is neglected, and T is a global time-dependent averaged surface temperature. Let the following equation be given for the energy balance model:

$$\overline{C}\frac{\partial T_A(t)}{\partial t} + A + BT_A(t) = \overline{S_{\text{sol}}}(t), \tag{1}$$

where $\overline{C}$ is the mean heat capacity, T_A is an approximation to the area-average of the time dependent temperature, A the CO_2 dependent radiative cooling, $B = 2.15$ the feedback effects such as water vapor cycles, lapse rate and cloud cover, $\overline{S_{\text{sol}}}(t)$ is the area-averaged solar forcing. To solve the zero-dimensional EBM you should proceed as described in the next steps.

1. Implement a function `calc_area`, which determines the area fraction of each grid cell as a function of latitude, with an equidistant step size $h = \frac{\pi}{64}$ for a 128×65 grid, and returns the values as a vector.

2. Implement a `calc_mean` function, using the results from the first task, which takes a matrix of values of a given parameter like albedo or solar forcing for each grid cell and returns the mean value over the whole area.

3. Write a function `calc_radiative_cooling_co2`, which returns the CO_2 dependent radiative cooling A for a given CO_2 concentration c. As default value use the CO_2 concentration of the year 1950: $c_0 = 315.0$. The radiative cooling A is given by

$$A(c) = 210.3 - 5.35 \log\left(\frac{c}{c_0}\right). \tag{2}$$

4. In a function `compute_equilibrium`, solve the above equation (1) using the forward Euler method and calculate an average temperature per year until equilibrium is reached.

5. Write a new function or adapt your existing one to add an option to solve the equation with the backward Euler method instead. Plot your results.

2 Control Solutions

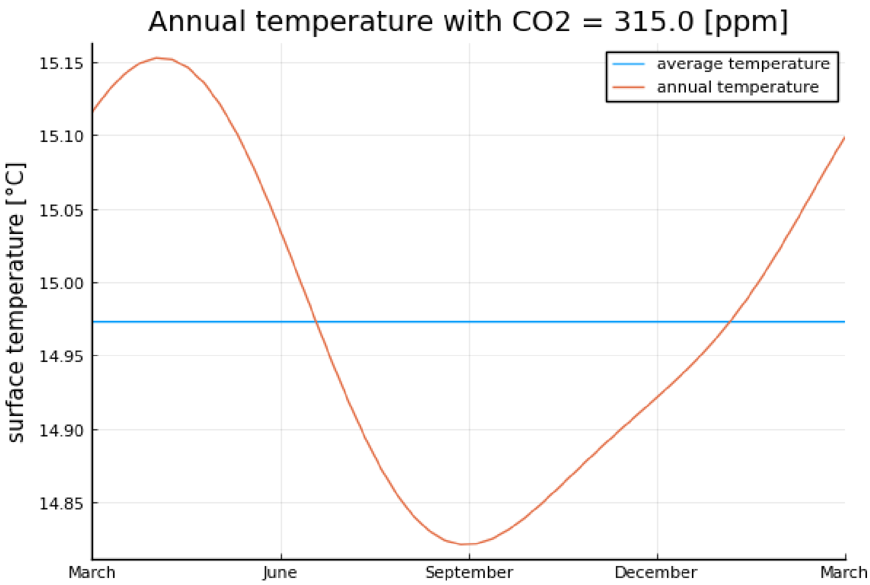

Figure 1: EBM results calculated with forward Euler method (results calculated with backward Euler method are similar)