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# An Introduction to Climate Modeling

**Milestone 6 - Fixing Stability in the spatial 2D Energy Balance Modell**

## 1 Fixing Stability in the spatial 2D Energy Balance Model

1. In the previous milestone you have implemented the spatial operator of the diffusive EBM for two spatial dimensions. The investigations of the eigenvalues revealed that the stable time step of the explicit Euler method is prohibitively small. Hence, we focus on the implicit Euler method. Adapt the function csmall`calc_equilibrium_2d` to use the backward Euler instead of the forward Euler time integration method. Visualize the results as an animation.
2. Use the functions `calc_area_mean`, `calc_area_mean_north` and `calc_area_mean_south` from previous milestones to compute the mean temperature values from the EBM in two space dimensions. Compare them with the results in milestone 4. Compare also the results for the temperature in Cologne. What is noticeable?
3. Write a function to compute equilibrium simulations based on NASA $CO_2$ data from 1980 to the present. Plot the results.
4. Now we want to investigate the sensitivity of the model to the parameters. Repeat the calculations of the temperature for the year 1950 with the parameters from Ziegler and Rehfeld 2021 [1].:

| Parameter | Zhuang et al. (2017b) | TransEBM v. 1.0 |
|---|---|---|
| Heat capacity $C$ [$10^6$ J K$^{-1}$ m$^{-2}$] | | |
| $C_{land}$ | 1.01 | 1.87 |
| $C_{ocean}$ | 288.4 | 394.47 |
| $C_{seaice}$ | 2.75 | 4.83 |
| $C_{snow}$ | 0.176 | 1.52 |
| $C_{atmos}$ | 4.72 | 0.79 |
| Albedo $a$ | | |
| $a_{land} + a_{landlat} P_2$ | $0.30 + 0.12 P_2 \in [0.24, 0.42]$ | $0.32 + 0.05 P_2 \in [0.295, 0.37]$ |
| $a_{ocean} + a_{oceanlat} P_2$ | $0.29 + 0.12 P_2 \in [0.23, 0.41]$ | $0.289 + 0.08 P_2 \in [0.249, 0.369]$ |
| $a_{seaice}$ | 0.60 | 0.60 |
| $a_{snow}$ | 0.75 | 0.70 |
| Thermal conductivity [W °C$^{-1}$] | | |
| $\kappa_{equ}$ | 0.65 | 0.90 |
| $\kappa_{ocean}$ | 0.40 | 0.40 |
| $\kappa_{land}$ | 0.65 | 0.65 |
| $\kappa_{land,NP}$ | 0.28 | 0.45 |
| $\kappa_{land,SP}$ | 0.20 | 0.12 |
| Outgoing radiation | | |
| $A$ [W m$^{-2}$] | 210.3 | 210.2 |
| $B$ [W m$^{-2}$ °C$^{-1}$] | 2.15 | 2.13 |
| $CO_{2,ref}$ [ppm] | 315 | 315 |
| $CO_{2,s}$ | 5.35 | 5.35 |

[1] E. Ziegler and K.Rehfeld, *TransEBM v. 1.0: description, tuning, and validation of a transient model of the Earth's energy balance in two dimensions*, Geoscientific Model Development 14.5, 2021.

Note that we have corrected the values for the first column to match the parameters used in this lecture and corrected the intervals for the albedo.

Compare your results with the temperatures calculated in 1.

5. Compute the equilibrium temperature from 1980 to present with the NASA $CO_2$ data as in task 3 but with the new parameter. Compare your results.

## 2 Control Solutions

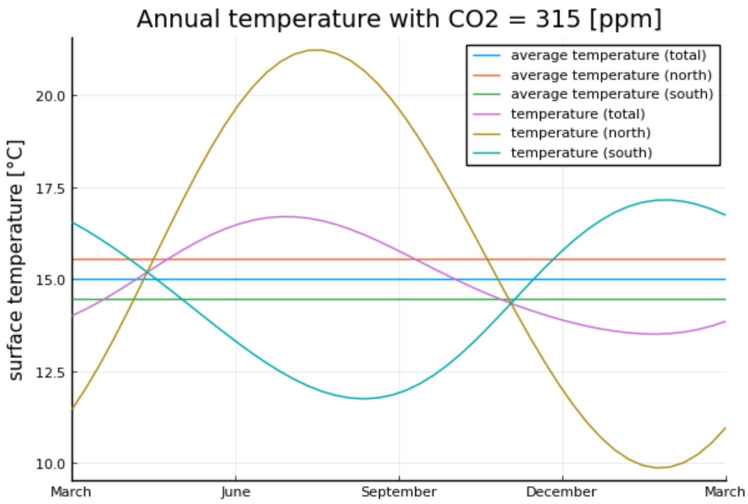


Figure 1: 2D EBM results for the mean temperatures

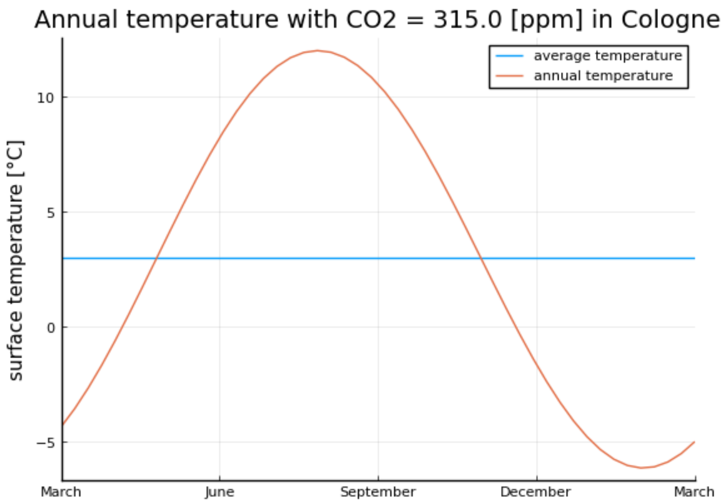


Figure 2: 2D EBM results for the temperature in Cologne

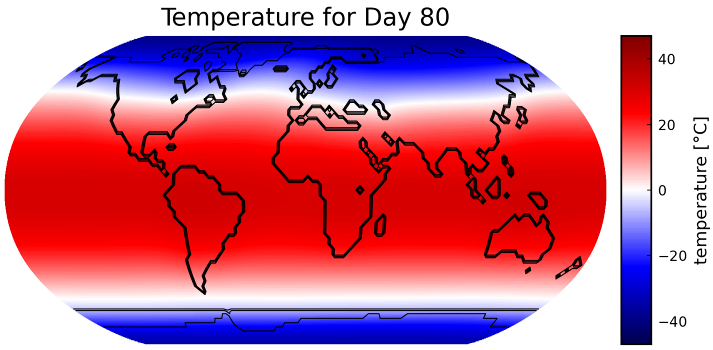


Figure 3: 2D EBM results for the temperatures on day 80

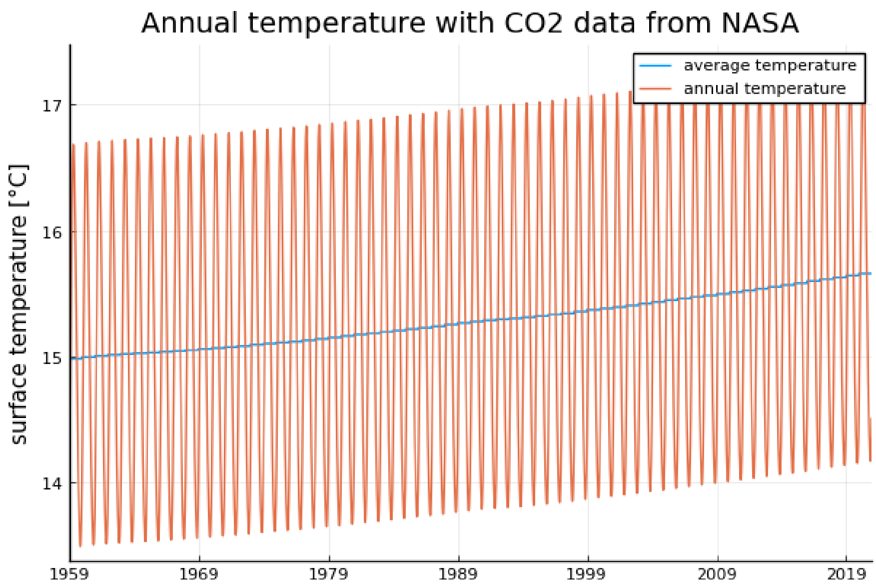


Figure 4: 2D EBM based on NASA data